TVM Problem Set 3: Intermediate Mortgage Finance
Self-Correcting, Hyperlinked File

Overview

The purpose of this problem set is to extend basic Time Value of Money (TVM) concepts to real estate financing. To lay a foundation, the problems are focused on residential property, although they can be easily extended to commercial investments. At this stage, it is assumed students:

- understand the basic "Six Functions of a $1,"
- can visualize how various TVM problems are set up,
- have intermediate skill on their financial analyst calculators,
- recognize the PV concept as a base decision tool for comparing options, and
- understand the basic concepts involved in residential finance.

Use of File

To use this file and its hyperlinks, read each question and try to work the answer. When you finish, or to sneak a look, click on the link to the answer. Read the steps and the solutions. To return to question, click the Return to Problem links. Note that the initial problems are set up with text support; the Addendum are patterned after them but focus on numbers.

Hyperlinked Table of Contents: Problem Statements

Problem 1. Residential Loan

This problem focuses on a basic residential mortgage transaction. Assume the following inputs:

Initial Assumptions	
Purchase Price	$220,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Problem 1 (a). Required Mortgage Payment

Under the initial assumptions, what is the monthly payment?

Click here for Answer 1 (a)

Problem 1 (b). Cost to borrower

In the previous problem, you calculated the monthly payment required to amortize the mortgage. What is the cost to the borrower of this loan arrangement (Note: cost should be interpreted as PV vs. actual nominal dollars)?

Click here for Answer 1 (b)

Problem 1 (c). Sensitivity to Rate

What are the required payments and the cost of the loan if the rate drops to 6.5% and all other terms are constant?

Assumptions	
Purchase Price	$220,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	6.50%
Borrower's Cost Cap.	10.00%

Click here for Answer 1 (c)

Problem 1 (d): Alternative House and Mortgage

As an alternative to the house in 1 a-c, assume that you found another house that had a higher price, but a more favorable financing package due to a tie-in between the lender and developer. What would the following cost you in terms of payments and present value?

Assumptions	
Purchase Price	$260,000
Loan-to-Value	90.00%
Periodicity	12
Term	30
Mortgage Rate	6.00%
Borrower's Cost Cap.	10.00%

Click here for Answer 1 (d)

Problem 1 (e). Comparing Loans: 30 year vs. 15 year

Assume that you have been shopping for loans and you have been offered a 30 year and a 15 year option. Using the following assumptions, compare the payments and the cost to you of the two alternatives. Based on numbers, which one is a better option and why?

Loan Option 1	
Purchase Price	$280,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$280,000
Loan-to-Value	80.00%
Periodicity	12
Term	15
Mortgage Rate	7.50%

Click here for Answer 1 (e)

Problem 1 (f). Starter Home

Assume you were just getting married and wanted to pick a starter home. You have two alternatives as outlined below. Which is a better choice and why?

Loan Option 1	
Purchase Price	$160,000
Loan-to-Value	90.00%
Periodicity	12
Term	30
Mortgage Rate	7.35%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$160,000
Loan-to-Value	80.00%
Periodicity	12
Term	15
Mortgage Rate	6.35%

Click here for Answer to 1 (f)

Problem 2. Prepayment

Problem 2 (a). Principal Balance

Assume that you anticipate moving out of your new home in 5 years. Under the terms of your initial mortgage, how much would you still owe the lender?

Initial Assumptions	
Purchase Price	$200,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Prepayment Assumptions

End of Year	5

Click here for Answer 2 (a)

Problem 2 (b). Cost of Prepaid Loan

Assuming you prepay your mortgage, what would the loan cost you as a borrower?

Click here for Answer 2 (b)

Problem 2 (c). Alternative Prepayment Problem

Using the same approach as in 2 a-b, what would you owe at prepayment and what would it cost you?

Initial Assumptions	
Purchase Price	$180,000
Loan-to-Value	80.00%
Periodicity	12
Term	20
Mortgage Rate	7.20%
Borrower's Cost Cap.	10.00%

Prepayment Assumptions

End of Year	3

Click here for Answer 2 (c)

Addendum: Additional Problems

A-1. Mortgage

In this case, you have been asked two questions; what is your mortgage payment, and what would that cost you in present value terms if you are the borrower? You might also compare the "cost" to the total outlay to see the interest portion of total payments.

Assumptions	
Purchase Price	$520,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	7.25%
Borrower's Cost Cap.	10.00%

Click here for A-1 Answer

A-2. Comparing Mortgage Terms

In this case, you are comparing two loans; a 30 year and a 20 year, with a different LV ration and different interest rates. Which of the two loans is "cheaper" to the borrower at their cost of capital?

Loan Option 1	
Purchase Price	$360,000
Loan-to-Value	70.00%
Periodicity	12
Term	30
Mortgage Rate	9.00%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$360,000
Loan-to-Value	80.00%
Periodicity	12
Term	20
Mortgage Rate	8.50%

Click here for A-2 Answer

A-3. Principal Balance

In this case, you are looking at the total cost to the borrower if they enter into a 30 year loan, but only hold it for 3 years. You will need to set up the problem in stages to calculate the monthly payment, to calculate the principal balance, and then to calculate the PV to the borrower of the monthly payment that is actually made plus the repayment of the principal balance outstanding at the end of the actual holding period of the loan.

Initial Assumptions	
Purchase Price	$180,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Prepayment Assumptions

End of Year	3

Click here for A-3 Answer

Answer 1 (a): Monthly Payment

Under the initial assumptions, what is the monthly payment?

Initial Assumptions	
Purchase Price	$220,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

The first step is to calculate the amount of the loan, and then plug the assumptions into your calculator. The initial principal is equal to the Price multiplied by the LV ratio.

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$176,000	
Payment	PMT		$1,291.43
Future Value	FV	$0	
Interest Rate	I	8.00%	

Click here to return to Problem 1 (a)

Answer 1 (b): Cost of Loan

In the previous problem, you calculated the monthly payment required to amortize the mortgage. What is the cost to the borrower of this loan arrangement (Note: cost should be interpreted as PV vs. actual nominal dollars)?

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV		$147,159
Payment	PMT	$ 1,291.43	
Future Value	FV	$0	
Interest Rate	I	10.00%	

Note that this differs from the total outlay, which would be:

Monthly	$1,291.43
Number	360
Total	$464,913

Click here to return to Problem 1 (b)

Answer 1 (c): Interest Rate Decline

What are the required payments and the cost of the loan if the rate drops to 6.5% and all other terms are constant?

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$176,000	
Payment	PMT		$1,112.44
Future Value	FV	$0	
Interest Rate	I	6.50%	

Monthly	$1,112.44
Number	360
Total	$400,478

Cost to Borrower

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV		$126,763
Payment	PMT	$ 1,112.44	
Future Value	FV	$0	
Interest Rate	I	10.00%	

Click here to return to Problem 1 (c)

Answer 1 (d): Alternative House

As an alternative to the house in 1 a-c, assume that you found another house that had a higher price, but a more favorable financing package due to a tie-in between the lender and developer. What would the following cost you in terms of payments and present value?

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$234,000	
Payment	PMT		$1,402.95
Future Value	FV	$0	
Interest Rate	I	6.00%	

Monthly	$1,402.95
Number	360
Total	$505,061

Cost to Borrower

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV		$159,867
Payment	PMT	$ 1,402.95	
Future Value	FV	$0	
Interest Rate	I	10.00%	

Click here to return to Problem 1 (d)

Answer 1 (e). Loan Options: 30 year vs. 15 year

Assume that you have been shopping for loans and you have been offered a 30 year and a 15 year option. Using the following assumptions, compare the payments and the cost to you of the two alternatives. Based on numbers, which one is a better option and why?

Loan Option 1	
Purchase Price	$280,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$280,000
Loan-to-Value	80.00%
Periodicity	12
Term	15
Mortgage Rate	7.50%

Calculations

		Loan Option 1		Loan Option 2	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	30		15	
Present Value	PV	$224,000		$224,000	
Payment	PMT		$1,643.63		$2,076.51
Future Value	FV	$0		$0	
Interest Rate	I	8.00%		7.50%	

Cost to Borrower

		Loan Option 1		Loan Option 2	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	30		15	
Present Value	PV		$187,293		$193,234
Payment	PMT	$ 1,643.63		$ 2,076.51	
Future Value	FV	$0		$0	
Interest Rate	I	10.00%		10.00%	
				Difference	$5,941

In terms of PV, the 30 year loan is better, with a PV savings of $5,941. Also, the 30 year loan has a lower monthly payment. In this situation, the choice would depend on the income of the borrower as well as attitudes toward risk and expectations of the future.

Click here to return to Problem 1 (e)

Answer 1-f: Starter Home

Assume you were just getting married and wanted to pick a starter home. You have two alternatives as outlined below. Which is a better choice and why?

Loan Option 1	
Purchase Price	$160,000
Loan-to-Value	90.00%
Periodicity	12
Term	30
Mortgage Rate	7.35%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$160,000
Loan-to-Value	80.00%
Periodicity	12
Term	15
Mortgage Rate	6.35%

		Loan Option 1		Loan Option 2	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	30		15	
Present Value	PV	$144,000		$128,000	
Payment	PMT		$992.12		$1,104.49
Future Value	FV	$0		$0	
Interest Rate	I	7.35%		6.35%	

Cost to Borrower

		Loan Option 1		Loan Option 2	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	30		15	
Present Value	PV		$113,053		$102,781
Payment	PMT	$ 992.12		$ 1,104.49	
Future Value	FV	$0		$0	
Interest Rate	I	10.00%		10.00%	
				Difference	($10,272)

Alternative	Eqyuity
Option 1	$16,000
Option 2	$32,000

Alternative	Equity
Option 1	$16,000
Option 2	$32,000

As noted in the comparisons, the 15 year loan is a better choice in terms of PV costs. However, it creates other problems in the sense that the mortgage payments are higher and, more importantly for many first-time buyers, the equity required is double due to the lower LV ratio. Many wouldn't be able to come up with the down payment and thus would be skewed to the 30 year loan.

Click here to return to Problem 1 (f)

Answer 2 (a): Prepayment

Assume that you anticipate moving out of your new home in 5 years. Under the terms of your initial mortgage, how much principal would you still owe the lender?

Initial Assumptions	
Purchase Price	$200,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Prepayment Assumptions	
End of Year	5

		Step 1: Payment		Step 2: Pb Remaining	
Factor	Code	Inputs	Answer	Inputs	Answer
Compounding/Period	m	12		12	
Term	t	30		25	
Present Value	PV	$160,000			$152,112
Payment	PMT		$1,174.02	$ 1,174.02	
Future Value	FV	$0		$0	
Interest Rate	I	8.00%		8.00%	

Click here to return to Problem 2 (a)

An swer 2 (b): Cost of Prepaid Loan

Assuming you prepay your mortgage, what would the loan cost you as a borrower?

Cost to Borrower

		Mortgage Payments		Principal Balance	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	5		5	
Present Value	PV		$55,256		$92,452
Payment	PMT	$ 1,174.02		$0	
Future Value	FV	$0		$152,112	
Interest Rate	I	10.00%		10.00%	
				Total PV	$147,708

$55,256 + $92,452

Click here to return to Problem 2 (b)

Answer 2 (c): Alternative Prepayment Problem

Using the same approach as in 2 (a)-(b), what would you owe at prepayment and what would it cost you?

		Step 1: Payment		Step 2: Pb Remaining	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	20		17	
Present Value	PV	$144,000			$133,195
Payment	PMT		$1,133.78	$ 1,133.78	
Future Value	FV	$0		$0	
Interest Rate	I	7.20%		7.20%	

Cost to Borrower

		Mortgage Payments		Principal Balance	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	3		3	
Present Value	PV		$35,137		$98,796
Payment	PMT	$ 1,133.78		$0	
Future Value	FV	$0		$133,195	
Interest Rate	I	10.00%		10.00%	
				Total PV	$133,933

Click here to return to Problem 2(c)

Addendum Answers

A-1 Answer. Mortgage Payment Answer

Assumptions	
Purchase Price	$520,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	7.25%
Borrower's Cost Cap.	10.00%

Factor	Code	Inputs	Answer			
Compounding/Period	m	12				
Term	t	30				
Present Value	PV	$416,000				
Payment	PMT		$2,837.85		Monthly	$2,837.85
Future Value	FV	$0			Number	360
Interest Rate	I	7.25%			Total	$1,021,627

Cost to Borrower

Factor	Code	Inputs	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV		$323,376
Payment	PMT	$ 2,837.85	
Future Value	FV	$0	
Interest Rate	I	10.00%	

Click here to Return to A-1 Question

A-2: Comparing Mortgage Terms Answer

Loan Option 1	
Purchase Price	$360,000
Loan-to-Value	70.00%
Periodicity	12
Term	30
Mortgage Rate	9.00%
Borrower's Cost Cap.	10.00%

Loan Option 2	
Purchase Price	$360,000
Loan-to-Value	80.00%
Periodicity	12
Term	20
Mortgage Rate	8.50%

		Loan Option 1		Loan Option 2	
Factor	Code	Inputs	Answer	Inputs	Answer
Compounding/Period	m	12		12	
Term	t	30		20	
Present Value	PV	$252,000		$288,000	
Payment	PMT		$2,027.65		$2,499.33
Future Value	FV	$0		$0	
Interest Rate	I	9.00%		8.50%	

Cost to Borrower

		Loan Option 1		Loan Option 2	
Factor	Code	Inputs	Answer	Inputs	Answer
Compounding/Period	m	12		12	
Term	t	30		20	
Present Value	PV		$231,052		$258,992
Payment	PMT	$ 2,027.65		$ 2,499.33	
Future Value	FV	$0		$0	
Interest Rate	I	10.00%		10.00%	
				Difference	$27,940

Alternative	Eqyuity
Option 1	$108,000
Option 2	$72,000

In comparing the two, note the differences between the costs in present value terms with the Loan 1 being "cheaper" than the second. That might be part of your selection, but note that it all depends on whether you can come up with the downpayment as noted below:

Alternative	Equity
Option 1	$108,000
Option 2	$72,000

Click here to Return to A-2 Question

A-3: Principal Balance Answer

Note the need to set the problem up in stages; to reconstruct the actual cash flows and then value them o the borrower.

Initial Assumptions	
Purchase Price	$180,000
Loan-to-Value	80.00%
Periodicity	12
Term	30
Mortgage Rate	8.00%
Borrower's Cost Cap.	10.00%

Prepayment Assumptions	
End of Year	3

		Step 1: Payment		Step 2: Pb Remaining	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	30		27	
Present Value	PV	$144,000			$140,083
Payment	PMT		$1,056.62	$ 1,056.62	
Future Value	FV	$0		$0	
Interest Rate	I	8.00%		8.00%	

Cost to Borrower

		Mortgage Payments		Principal Balance	
Factor	**Code**	**Inputs**	**Answer**	**Inputs**	**Answer**
Compounding/Period	m	12		12	
Term	t	3		3	
Present Value	PV		$32,746		$103,905
Payment	PMT	$ 1,056.62		$0	
Future Value	FV	$0		$140,083	
Interest Rate	I	10.00%		10.00%	
				Total PV	$136,651

Click here to Return to A-3 Question

Click here to go to Beginning of Addendum

Click here to go to Start