TVM Problem Set 1: Self-Correcting, Hyperlinked File

Purpose of Problem Set

The purpose of this problem set is to present you with a series of Time Value of Money (TVM) problems that you can use to ensure that you have an intuitive understanding of the underlying concepts and can solve basic problems. To that end, the problem set is self-contained with both the questions and answers embedded in hyperlinks. These problems can be answered by using the Six Functions of $1 tables, a financial calculator, the raw equations, or Excel's built-in financial functions. With the exception of the FV1, all payments/receipts are at the end of the period as is the default setting in the tables.

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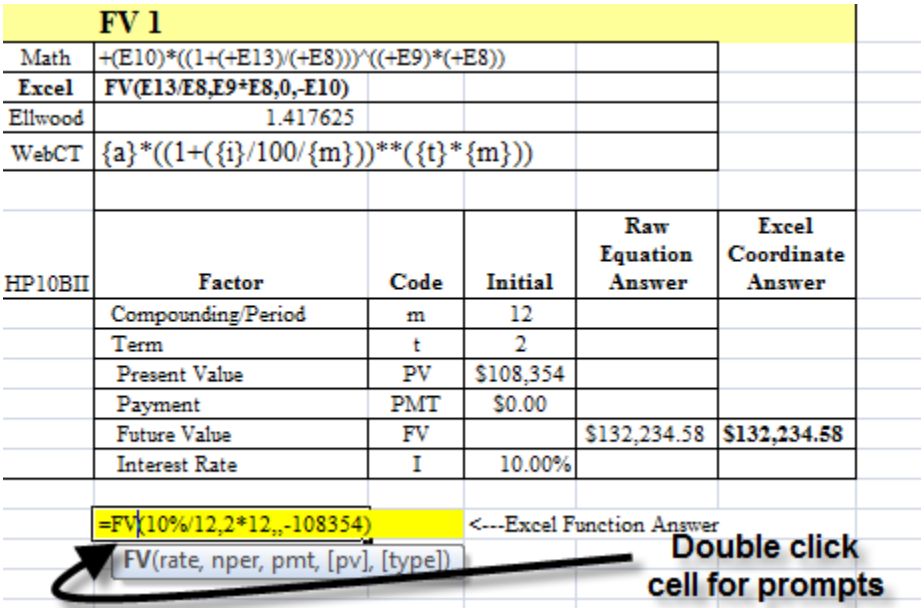
Overview

Excel/HP Integration

This worksheet is to be used in conjunction with the TVM Problem Set 1 Excel Model named: 6F_ToolboxProbSet1v1. You will see two sheets; 6 functions which has the following structure for each of the 6 functions, and Named TVMs in which the Excel equations uses variable names (e.g., FV, PV). Even if you can work the problems, it is suggested you take a look at the answers which show you alternative ways of calculating them in Excel. This is the format for each function:



Use of File

To use this file and its hyperlinks, read each question, it is suggested that you try to compute the answer. When you finish your attempt --or want to sneak a look at the answer-- click on "Click here for Answer" link. Note, you may have to enter CTRL-click simultaneously. Read the steps and the solutions. To return to question, click the Return to Problem links.

Problems

Problem 1. Gift

Assume you have received a gift of $12,000. If you deposit the money in a relatively safe investment earning 6% annually, compounded monthly, how much would it grow to in 10 years?

Click here for Answer 1

Problem 2. Gift over Time

Problem 2 (a). As an alternative to the outright gift of $12,000, your relative promised to pay you $1,400 each year at the end of the period for 10 years, for a total of $14,000. If you have the ability to earn 8% compounded annually on your money, from a pure economic perspective, which choice is right for you?

Click here for Answer 2-a

Problem 2 (b). What is the minimum you would you have to get from your relative each year to get you to accept the annuity in Problem 2 over the lump $12,000? In this case, assume you don't fully "trust" your relative and think you should get a 2% premium on your return to justify deferring payment.

Click here for Answer 2-b

Problem 3. Earning a Targeted Sum

Problem 3 (a). Assume that you want to buy a new car when you graduate in 5 years (yes, you've been inspired to get your Ph.D.). Assuming your dream car costs $60,000 and you can earn 8% annual, compounded monthly, how much would you have to put aside each month to be able to acquire your car outright when you graduate?

Click here for Answer 3 (a)

Problem 3 (b). As an alternative, what lump sum would you have to deposit today to cover the cost of your $60,000 car assuming your dream car costs $60,000 and you can earn 8% annual, compounded monthly?

Click here for Answer 3 (b)

Problem 3 (c). Earning a Moving Target. In the previous problems, we ignored the fact that the price of your dream car would also escalate. Assume that we go into a high, sustained period of inflation and that the price of your $60,000 car goes up 6% per year, compounded monthly. How much would you have to put aside each month earning your 8% annual, compounded monthly?

Click here for Answer 3 (c)

Problem 4. Student Investment

Assume that you are currently working in a job full time and are considering going back to school to get a graduate degree so that you can advance your career. However, you have been in the workforce for a number of years, and anticipate working only 8 more years after you get your degree before you retire. Your cost of capital or interest earning potential is 10% annual, compounded monthly, since you are fairly aggressive in managing your portfolio. You currently earn $60,000 per year, paid monthly. Answer the following questions.

Problem 4 (a). Assume you want to "get it over with," and are going to drop out of work and then come back in refreshed, recharged and retooled. You're used to hard work and think you can finish your degree in 2 years, including some downtime for a vacation before you get on with your life. Furthermore, your employer values your contribution so much, it is willing to fund your sabbatical and cover your educational costs, but will require you to take a leave without pay. How much more would you have to earn on an annual basis (ignore raises, fringes and other soft items) to justify going back to school if your sole reason is to improve your wealth position?

Click here for Answer 4 (a)

By this time, you should be ready to develop a better understanding of how your calculator stores numbers in registers so you can do sensitivity analysis. Think about what minimal inputs you would have to change to determine what impact various assumptions would make on your answer. For example, without using your calculator, think about what would happen if you:

1) raise your salary to $80,000 (congratulations),
2) lower the interest rates, or
3) make you younger or extend your retirement so you have 18 years to work when you return.

Problem 4 (b). Rather than "getting it over with," you could opt for part-time schooling, which would take you 4 years to complete. During those 4 years you will earn the same as you do today (sorry, no raises), but again, your company pays for your education. How much would you have to earn over the remaining 6 years (in 10 you retire) to justify going back to school part time? Your cost of capital or interest earning potential is still 10% annual, compounded monthly.

Again, you currently earn $60,000 per year, paid monthly and your employer will pay your full costs of education.

Click here for Answer 4 (b)

Problem 4 (c). Now, let's make this question a little more interesting. Say you believe that you could get a 30% raise if you stay with your current employer (indeed, that's their policy for rewarding knowledge workers). Ignoring other factors and assuming they pay your educational expenses, what is the value of your educational "investment?"

Click here for Answer 4 (c)

Problem 4 (d). What if you're younger, and have a 20 year work/school horizon?

Click here for Answer 4 (d)

Problem 4 (e). Now, what if your cost of capital is a more conservative 6% and you have 20 working years ahead?

Click here for Answer 4 (e)

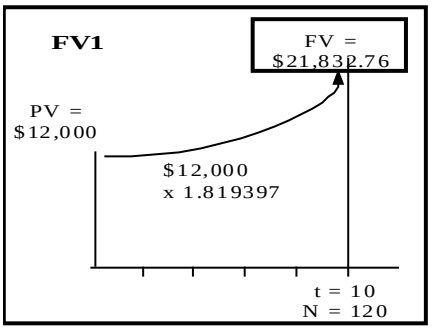
Answers

Answer Problem 1.

Assume you have received a gift of $12,000. If you deposit the money in a relatively safe investment earning 6% annually, compounded monthly, how much would it grow to in 10 years?

This is a **FV1** problem.

A. Visual/Table Approach (note: the table uses FV factors which are multiplied by the PV)



B. Calculator Approach (Using the HP 10BII)

- First, clear all by using the GOLD key and then the C ALL key.
- Second, tell the HP that you want to work monthly by entering 12, pressing the GOLD key, and then P/YR.
- Third, enter 10, press the GOLD key, and then press xP/YR (N: note this adjusts N for payments per year. Since it is annual, it loads 10; if monthly, would be 120)
- Fourth, enter 6, press the GOLD key, and then press NOM%.
- Fifth, enter 12,000 and press PV.
- Press FV to solve and note $21,832.76 after rounding to 2 decimals.

C. Excel Approach

This question can be answered using the Table below or the FV function. Note in the FV calculation, both the RATE and the NPER (number of periods of compounding) are converted to monthly equivalents. This can be done in the equation (i.e., 6%/12) or fed into it (i.e., 10 * 12 = 120). Please note also that this equation also handles the FV of an annuity which would be accomplished by feeding the PMT with a value and leaving PV blank. Finally, the PV goes in as a negative to yield a positive answer since you would be receiving the FV and paying in the PV.

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	10	
Present Value	PV	$12,000	
Payment	PMT	$0.00	
Future Value	FV		$21,832.76
Interest Rate	I	6.00%	

=FV(6%/12,120,,-12000)

FV(rate, nper, pmt, [pv], [type])

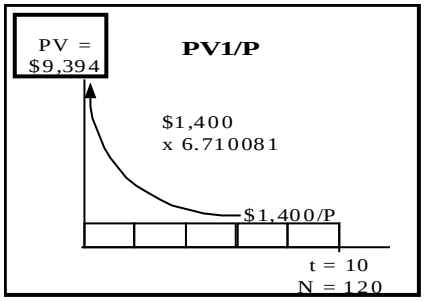
Click here to return to Problem 1

Answer Problem 2 (a)

As an alternative to the outright gift of $12,000, your relative promised to pay you $1,400 each year at the end of each year for 10 years, for a total of $14,000. If you have the ability to earn 8% annual on your money, from a pure economic perspective, which choice is right for you?

This is a **PVA** or Present Value of an Annuity Problem.

A. Picture/Table Factor Approach (Note: you should use the Annual table)



B. Calculator Approach (Using the HP 10BII)

- First, clear all by using the GOLD key and then the C ALL key.
- Second, tell the HP that you want to work annually by entering 1, pressing the GOLD key, and then P/YR.
- Third, enter 10, press the GOLD key, and then press xP/YR (N)
- Fourth, enter 8, press the GOLD key, and then press NOM%.
- Fifth, enter 1,400 and press PMT.
- Press PV to solve and note $9,394.11.

C. Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	1	
Term	t	10	
Present Value	PV		$9,394.11
Payment	PMT	$1,400	
Future Value	FV	$0	
Interest Rate	I	8.00%	

=PV(8%,10,-1400)

PV(rate, nper, pmt, [fv], [type])

Click here to return to Problem 2 (a)

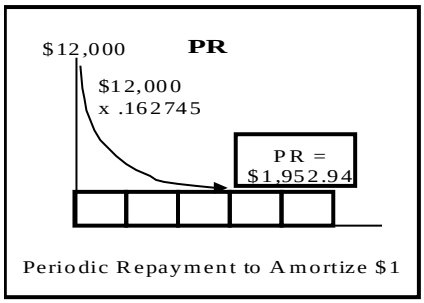
Answer Problem 2 (b)

What is the minimum you would you have to get from your relative each year for 10 years to get you to accept the annuity in Problem 2 over the lump $12,000? In this case, assume you don't fully "trust" your relative and think you should get a 2% premium on your return to justify deferring payment.

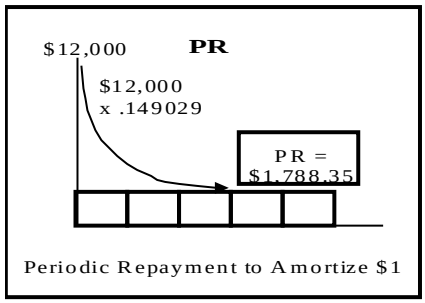
This is a **PR** or **PMT** problem.

A. Picture/Table Factor Approach

This is the answer with 10% risk-adjusted.



This is the answer if 8% and not adjusted for risk.



B. Calculator Approach (Using the HP 10BII)

- First, clear all by using the GOLD key and then the C ALL key.
- Second, tell the HP that you want to work annually by entering 1, pressing the GOLD key, and then P/YR.
- Third, enter 10, press the GOLD key, and then press xP/YR (N)
- Fourth, enter 10, press the GOLD key, and then press NOM%.
- Fifth, enter 12,000 and press PV.
- Press PMT to solve and note $1,952.95.

Factor	Code	Initial	Answer
Compounding/Period	m	1	
Term	t	10	
Present Value	PV	$12,000	
Payment	PMT		$1,952.94
Future Value	FV	$0	
Interest Rate	I	10.00%	

=PMT(10%,10,-12000)

PMT(rate, nper, pv, [fv], [type])

← Risk-adjusted

| Not risk-adjusted

Factor	Code	Initial	Answer
Compounding/Period	m	1	
Term	t	10	
Present Value	PV	$12,000	
Payment	PMT		$1,788.35
Future Value	FV	$0	
Interest Rate	I	8.00%	

=PMT(8%,10,-12000)

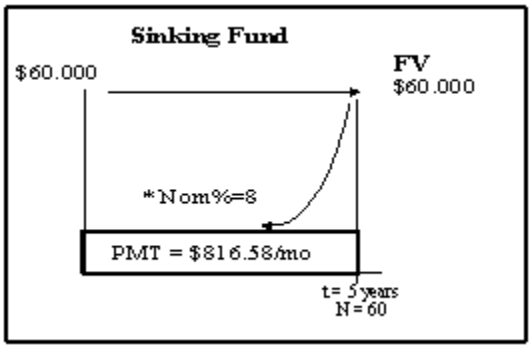
PMT(rate, nper, pv, [fv], [type])

Click here to return to Problem 2 (b)

Answer 3 (a): Earning a Targeted Sum with an Annuity

3 (a). Assume that you want to buy a new car when you graduate in 5 years (yes, you've been inspired to get your Ph.D.). Assuming your dream car costs $60,000 and you can earn 8% annual, compounded monthly, how much would you have to put aside each month to be able to acquire your car outright when you graduate?

This is a **SF** Sinking Fund problem; it is the reciprocal of a PR problem and uses the PMT function.



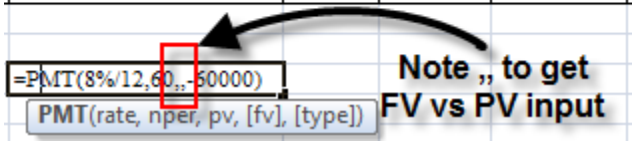
Given the nature of the problem, Excel uses the same core equation as with the PMT calculation. Be careful to input the value in the appropriate place in the equation. Excel assists with highlighting the variables on the help line as you move through the equation.

A. Abbreviated Calculator Approach (Using the HP 10BII)

HP10BII: P/YR=12, x/YR:N=5, NOM%=8, FV=60,000, Press PMT (Answer: $816.58)

B. Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	5	
Present Value	PV	$0	
Payment	PMT		$816.58
Future Value	FV	$60,000	
Interest Rate	I	8.00%	

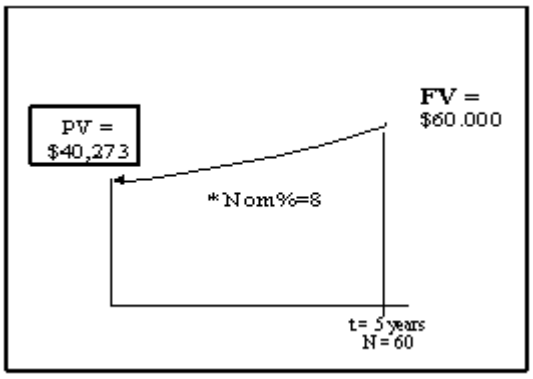


Click here to return to Problem 3

Answer 3 (b): Earning a Targeted Sum with a Lump

3 (b). As an alternative, what lump sum would you have to deposit today to cover the cost of your car assuming your dream car costs $60,000 and you can earn 8% annual, compounded monthly for a 5 year hold?

This is a **PV** problem.



A. Abbreviated Calculator Approach (Using the HP 10BII)

HP10BII: P/YR=12, x/YR:N=5, NOM%=8, FV=60,000, Press PV (Answer: 40,273)

B. Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	5	
Present Value	PV		$40,272.63
Payment	PMT	$0.00	
Future Value	FV	$60,000	
Interest Rate	I	8.00%	

=PV(8%/12,5*12,,-60000)

PV(rate, nper, pmt, [fv], [type])

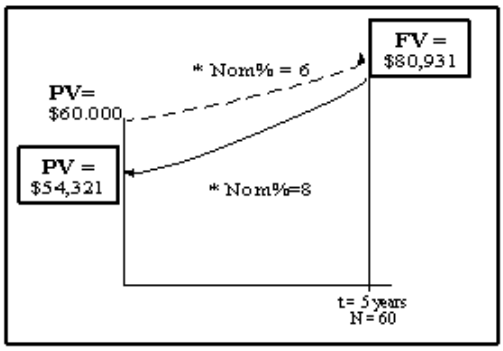
Note ,, to get to FV variable

Click here to return to Problem 3 (b)

Answer 3 (c): Earning a Moving Target

3 (c). Earning a Moving Target. In the previous problems, we ignored the fact that the price of your dream car would also escalate. Assume that we go into a high, sustained period of inflation and that the price of your $60,000 car goes up 6% per year, compounded monthly. How much would you have to put aside today earning your 8% annual, compounded monthly?

This is a hybrid or two-step problem: first, calculate **FV**, then calculate **PV** of that FV.



A. Abbreviated Calculator Approach (Using the HP 10BII)

Step 1 HP10BII: P/YR=12, xP/YR:N=5, Nom%=6, PV=60,000, Press FV (Answer: 80,931)
Step 2 HP10BII: P/YR=12, xP/YR:N=5, Nom%=8, FV=80,931, Press PV (Answer: 54,321)

B. Excel Approach

Step 1: Calculate FV

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	5	
Present Value	PV	$60,000	
Payment	PMT	$0.00	
Future Value	FV		$80,931.01
Interest Rate	I	6.00%	

=FV(6%/12,5*12,,-60000)

FV(rate, nper, pmt, [pv], [type])

Step 2: Calculate PV

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	5	
Present Value	PV		$54,321.74
Payment	PMT	$0.00	
Future Value	FV	$80,931	
Interest Rate	I	8.00%	

=PV(8%/12,5*12,,-80931)

PV(rate, nper, pmt, [fv], [type])

Click here to return to Problem 3 (c)

Answer Problem 4 (a)

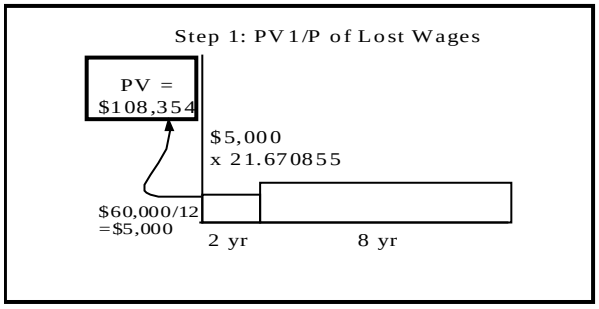
Assume you want to "get it over with," and are going to drop out of work and then come back in refreshed, recharged and retooled. You're used to hard work and think you can finish your degree in 2 years, including some downtime for a vacation before you get on with your life. Furthermore, your employer values your contribution so much, it is willing to fund your sabbatical and cover your educational costs, but will require you to take a leave without pay. How much more would you have to earn on an annual basis (ignore raises, fringes and other soft items) to justify going back to school if your sole reason is to improve your wealth position?

This is a multi-stage problem using PV, FV and PMT. You can break this problem down into three stages.

A. Abbreviated Calculator Approach (Using the HP 10BII)

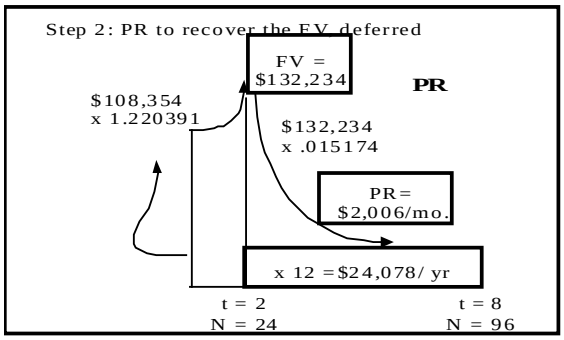
In Step 1, figure out the PV of the lost wages for your two year sabbatical. You should come up with $108,354, assuming the monthly payment is $5,000 ($60,000 wage/12 months):

HP10BII: P/YR=12, xP/YR:N=2, NOM%=10, PMT=5,000, Press PV



In Step 2, you carry the PV lost forward to the FV that you will have to recover to offset your PV of lost wages. Step 2 (a): Calculate FV of lost wages to establish deferred target:

HP10BII: P/YR=12, xP/YR:N=2, NOM%=10, PV=108,354, Press FV



In Step 3, calculate Periodic Repayment factor to amortize:

- HP10BII: P/YR=12, xP/YR:N=8, Nom%=10, PV=$132,234, Press PMT and multiply by 12
- Answer: $24,078/year required income bonus.

B. Excel Approach

Step 1: PV of Lost Wages

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	2	
Present Value	PV		$108,354.27
Payment	PMT	$5,000	
Future Value	FV	$0	
Interest Rate	I	10.00%	

=PV(10%/12,2*12,-5000)

PV(rate, nper, pmt, [fv], [type])

Step 2: FV of PV of Lost Wages

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	2	
Present Value	PV	$108,354	
Payment	PMT	$0.00	
Future Value	FV		$132,234.58
Interest Rate	I	10.00%	

=FV(10%/12,2*12,,-108354)

FV(rate, nper, pmt, [pv], [type])

Step 3: Calculate Income Premium/mo. to recover FV of PF of lost Wages (PR)

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	8	
Present Value	PV	$132,234	
Payment	PMT		$2,006.54
Future Value	FV	$0	
Interest Rate	I	10.00%	

=PMT(10%/12,8*12,-132234) 12 * $2,006.54

PMT(rate, nper, pv, [fv], [type]) $ 24,078

Discussion of Problem 4 (a)

There are a number of ways to arrive at the same answer. For example, you could calculate the FV of the lost payments directly, and then complete Step 3 as outlined above. You might wonder

why everything isn't brought back to the present. While that would be the usual procedure, it's not necessary here since you are only looking at the premium payment to recover the $132,234 which is deferred 2 years and when brought back to today = $108,354 which is the PV of the lost wages of $5,000. Think about it, and you should see how it is really a PV problem after all. This case should help you understand the importance of visualizing the problem you are trying to solve.

Click here to return to Problem 4 (a)

Answer 4 (b)

Sensitivity Analysis. Now, let's look at how a financial calculator and Excel can help us do some basic sensitivity analysis without clearing registers each time. For example, in establishing the PV of the lost $5,000 payment for 24 months, we entered the following:

HP10BII: P/YR=12, xP/YR:N=2, Nom%=10, PMT=$5,000; PV (Answer: $108,354)

Step 1: Options for Stage 1.

- Salary Change. What if we changed your salary to $80,000? What would you have to do to solve this part of the problem? If you think about it, the only thing you are changing is the PMT which goes from $5,000 to $6,667 ($80,000/12month).
  - HP. Assuming you have the basic assumptions loaded to the HP, Since everything else is the same, just enter the $6,667 to the PMT and press the PV key to get $144,480. To see if this really works, key $5,000 into PMT, press PV and you get back to where you started.
  - Excel. As with HP, just change the Payment

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	2	
Present Value	PV		$144,479.59
Payment	PMT	$6,667	
Future Value	FV	$0	
Interest Rate	I	10.00%	

=PV(10%/12,2*12,-6667)

PV(rate, nper, pmt, [fv], [type])

- Interest Rate Changes: Stage 1
  - HP. The same is true with the interest rate. Assuming you entered the $5,000 to PMT, now lower the interest rate to 8%. You get $110,553. Why are you getting a higher present value with a lower rate? This relationship is important: the higher the rate, the lower the PV; the lower the rate, the higher the PV.
  - Excel. As with the HP, just change Rate (also, change PMT back to $5,000)

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	2	
Present Value	PV		$110,552.72
Payment	PMT	$5,000	
Future Value	FV	$0	
Interest Rate	I	8.00%	

=PV(8%/12,2*12,-5000)

PV(rate, nper, pmt, [fv], [type])

Step 2: Options for Stage 2.

- Number of Future Working Years. The same basic logic applies to the Step 2 calculations; once you enter your 3 independent values, you can change any one and solve for the new dependent value. For example, assume that the $132,234 is still your lost FV of the earnings. However, rather than 8 years, assume that you are going to work for another 18 years.

  - HP Solution.
    - You got your base case monthly premium by entering:

    HP10BII: P/YR=12, xP/YR:N=8, Nom%=10,PV=$132,234, Press PMT (Answer : $2,006/month)

    - Now, merely enter 18 for xP/YR:N instead of 8, and solve for your PMT, and you get $1,322 as the required monthly raise to make you whole. This translates to $15,866/year (i.e., 12*$1,322)

  - Excel Solution

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	18	
Present Value	PV	$132,234	
Payment	PMT		$1,322.13
Future Value	FV	$0	
Interest Rate	I	10.00%	

=PMT(10%/12,18*12,-132234) 12 * $1,322.13

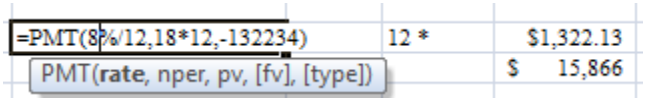
PMT(rate, nper, pv, [fv], [type]) $ 15,866

- What if Lower Rate and Longer Working Period? Assume that you are moving into a stage of life where your risk tolerances are lower and you lower your required return to 8% for the 18 year expected working period?

  - HP Solution.
    - Previous Problem with 18 working years was:

    HP10BII: P/YR=12, xP/YR:N=88, Nom%=10,PV=$132,234, Press PMT (Answer : $1,322/month)

    - Now, lower your earning requirement on this new PMT by substituting 8% for Nom%, and get $1,157.
  - Excel Solution



Click here to return to Discussion of Problem 4 (a)

Answer 4 (b)

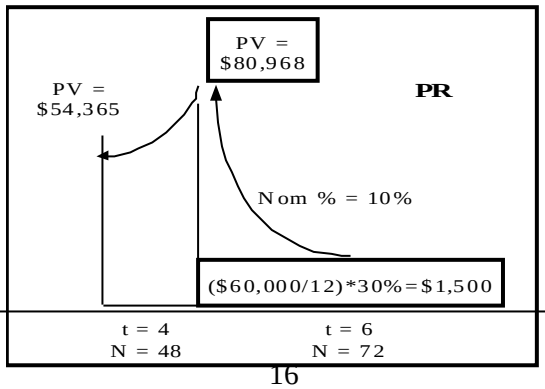
Rather than "getting it over with," you could opt for part-time schooling, which would take you 4 years to complete. During those 4 years you will earn the same as you do today (sorry, no raises), but again, your company pays for your education. How much would you have to earn over your current salary over the remaining 6 years (in 10 you retire) to justify going back to school part time? Your cost of capital or interest earning potential is still 10% annual, compounded monthly. Again, you currently earn $60,000 per year, paid monthly and your employer pays for your education as a job benefit.

Surprise; ignoring the fact that "time is money," you don't have to earn anything more since there is no outlay on your part.

Click here to return to Problem 4 (b)

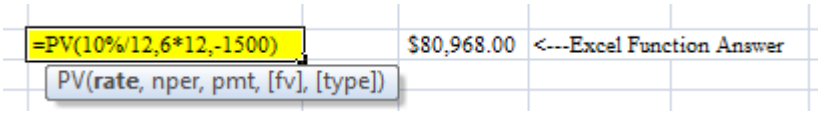
Answer 4 (c)

Now, let's make this question a little more interesting. Say you believe that you could get a 30% raise if you stay with your current employer (indeed, that's their policy for rewarding knowledge workers). Ignoring other factors and assuming they pay your educational expenses, what is the value of your educational "investment?"

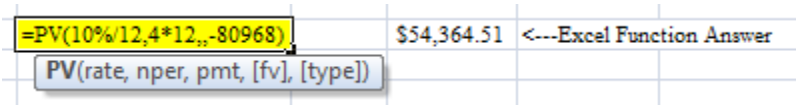


- HP Solution.
  - Step 1 HP10BII: P/YR=12, X P/YR=6, Nom%=10,PMT=1,500, Press PV (Answer: 80,968)
  - Step 2 HP10BII: P/YR=12, X P/YR=4, NOM%=10, FV=80,968, Press PV (Answer: 54,365)

- Excel Solution
  - Step 1



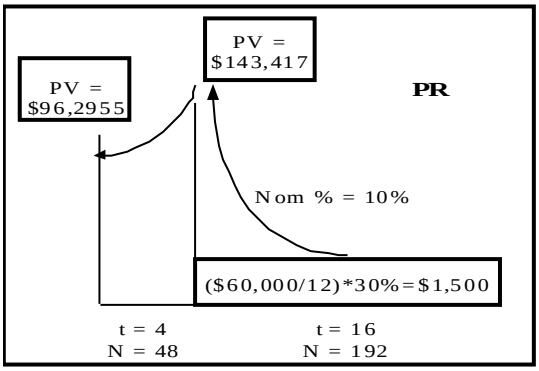
  - Step 2



Click here to return to Discussion of Problem 4 (c)

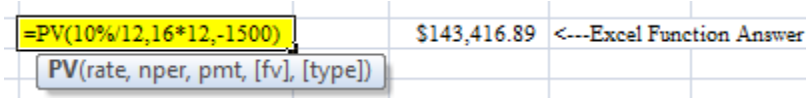
Answer 4 (d)

What if you're younger, and have a 20 year work/school horizon and are expecting the 30% raise and have a 10% cost of capital?

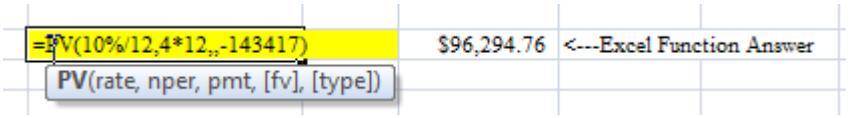


- HP Solution.
  - Step 1 HP10BII: P/YR=12, X P/YR=16, Nom%=10,PMT=1,500,Press PV(Answer: 143,417)
  - Step 2 HP10BII: P/YR=12, X P/YR=4, NOM%=10, FV=80,968, Press PV (Answer: 96,295)

- Excel Solution
  - Step 1



- Step 2

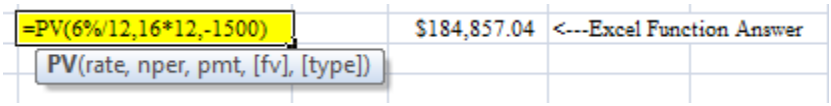


Click here to return to Discussion of Problem 4 (d)

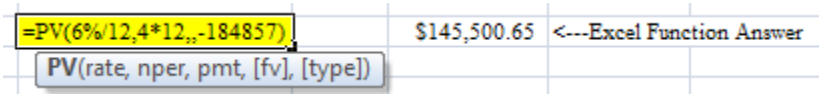
Answer 4 (e)

Now, what if your cost of capital is a more conservative 6%, and you have 20 years ahead?

- HP Solution.
  - Step 1 HP10BII: P/YR=12, X P/YR=16, Nom% =6,PMT=1,500,Press PV(Answer: 184,857)
  - Step 2 HP10BII: P/YR=12, X P/YR=4, NOM%=6, FV=184,857, Press PV(Answer:145,501)
  -
- Excel Solution
  - Step 1



  - Step 2



The 4-year deferred value of the $1,500 is $184,857
The present value of that deferral is $145,501

Click here to return to Discussion of Problem 4 (e)

Click here to Start Over at Beginning of Problem Set.