TVM Problem Set 2: Introduction to Mortgage Finance

Self-Correcting, Hyperlinked File

Overview

The purpose of this problem set is to present you with a series of Time Value of Money (TVM) problems that address various aspects of mortgage financing, ranging from simple fixed rate mortgage calculations, to more complicated financially engineered loan arrangements. The problems are somewhat incremental, beginning with basic concepts and then extending to more elaborate financial structures. To master this level, it is assumed that students have developed: 1) a basic understanding of the mechanics of TVM, 2) the ability to visualize or intuitively model cash inflows and outflows, 3) an adequate comfort level with their chosen calculators, and 4) an understanding of some of key financial principles embedded in mortgage finance. It should be noted that the solutions presented herein are both visual and mechanical, employing the 5F's that we introduced in class: Fill Four Factors, Find the Fifth.

To use this file and its hyperlinks as a learning tool, read each question and try to work the answer. When you finish, or need a hint, click on the link to the answer. To return to question, click the Return to Question links.

Table of Contents

Overview 1
Questions 1
Problem 1. Traditional Mortgage 1
a. Entry-level House 1
b. Sensitivity Analysis 1
Problem 2. Alternative Interest Rates and Terms 3
a. Relocation 3
Problem 3. Alternative Financing 5
Alternative Financing 5
Answers 7
Answer Problem 1 (a) 7
A. Problem Visualization 7
B. Using the HP 10BII 7
Problem 1 (b) Sensitivity Analysis 9
Using the HP 10BII 9
Using the HP 10BII 9
Using the HP 10BII 10
Using the HP 10BII 11
Using the HP 10BII 13
Using the HP 10BII 14
Problem 2 15
Using the HP 10BII 15
Using the HP 10BII 16
Using the HP 10BII 18
Using the HP 10BII 19

Questions

Problem 1. Traditional Mortgage

a. Entry-level House

Assume you are seeking to buy an "entry-level" house that is available from the builder for $200,000. Currently, you can get an 80% loan, at 7.35% with a 30 year amortization. How much could you borrow? What would your monthly mortgage payments be?

Click here for Answer 1 (a)

b. Sensitivity Analysis

Now let's run through some calculations and solve for the correct answers. (Note: for each question, go back to initial assumptions for other variables).

(1) Interest rates fall to 6%, what are mortgage payments?

Click here for Answer 1-b (1)

(2) You add $20,000 for landscaping and other amenities and build that into the price of the house. What is your mortgage payment?

Click here for Answer 1-b (2)

(3) The term of the loan is reduced to 25 years. What is your mortgage payment?

Click here for Answer 1-b (3)

(4) Assume you sell the house after 5 years. How much will you still owe on the initial mortgage?

Click here for Answer 1-b (4)

(5) Assume your house appreciates 6% per year, compounded monthly. How much equity will you walk away with if you sell after 10 years? (Note: ignore selling expenses)

Click here for Answer 1-b (5)

(6) If your cost of capital is 8%, what did the deal in 1-b (5) actually cost you in PV?

Click here for Answer 1-b (6)

Problem 2. Alternative Interest Rates and Terms

a. Relocation

Assume that in this case, you have decided to move out to a northern county in which the housing prices are more "affordable," and have found your dream house for a mere $180,000. Assuming you can get a loan for 80% of the value for 30 years, and interest rates are still 7.35%. What would your mortgage payments be per month?

Click here for Answer 2-a

b. As of January 24, 2001, the following mortgage terms were available as of January 25, 2001.

Today's National Average Mortgage Rates from HSH

Program	Rate	Points
30 Year FRM	7.35%	0.85%
15 Year FRM	6.90%	0.82%
1 Year ARM	6.59%	0.88%

(1). What are the initial monthly payments under the 15 year and 1 year ARM options (ignore the points)? Note: the ARM is a 30 year amortization.

Click here for Answer 2-b (1)

(2). Compare the "costs" of the 30 year and the15 year mortgages above. Which one is "more affordable?" Assume your cost of capital is 10% annual, compounded monthly.

Click here for Answer 2-b (2)

(3) Let's build on the prior question. What rate would the 15 Year loan have to be at to make you indifferent in terms of the present value cost vs. the 30 year loan? Refer to the answer to 2-b (2) to make sure you have the right inputs. (Note that this problem is intended to stretch your thinking a bit. The 5-Fs should help you figure out how to solve for the rate; also, to get a rate, you need to compare some PV (or FV) to some PMT so you have a meaningful comparison.)

Click here for Answer 2-b (3)

Problem 3. Alternative Financing

Alternative Financing

To simplify the discussion, we will use the same basic problem presented in Problem 2. That is, your dream house costs $180,000, you can get a loan for 80% of the value for 30 years, and interest rates are still 7.35% annual, compounded monthly.

(1) Now, let's go back to your original assumptions. Assume that you can only afford to make $900 in monthly payments. The good news for you is that the housing market is softening and the builder is anxious to get you into the new house. How much would he have to "buy-down" your mortgage amount to allow you to afford the 30 year, fully amortizing loan at 7.35%? (Hint: try to sketch this out before you solve it)

Click here for Answer 3-a (1)

Why would a builder use a rebate rather than just cutting the price of the house; wouldn't that be simpler? This is a "thought" question; no calculations required.

Click here for Answer 3-a (2)

(3) Assume that the builder had a construction loan that carried a 200 basis point spread (i.e., 2%) over the rate on a fully amortizing, 30 year permanent loan. If he had to carry the house for 2 years until the market would absorb the house, how much more would he have to charge then to break even? (Note: the builder has financed 90% of the actual costs and builds a 15% margin into the price). He also considers his cost of capital 15%.

Click here for Answer 3-a (3)

(4) Let's say the builder didn't want to carry the property to wait out the market, what discount would that translate to if the builder decided to cut the cost today to move the property?

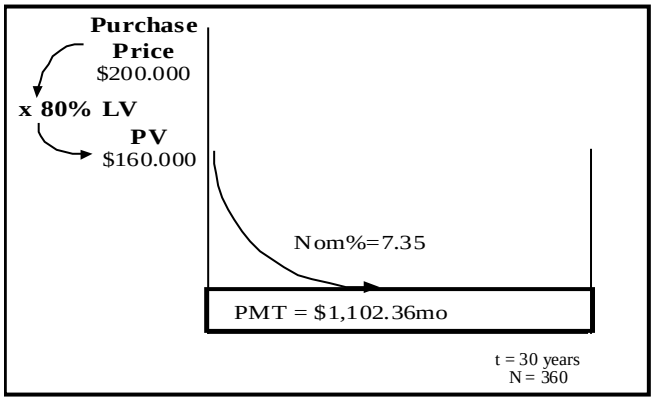
Click here for Answer 3-a (4)

Answers

Answer Problem 1 (a)

Assume you are seeking to buy an "entry-level" house that is available from the builder for $200,000. Currently, you can get an 80% loan, at 7.35% with a 30 year amortization. How much could you borrow? What would your monthly mortgage payments be?

A. Problem Visualization



B. Using the HP 10BII

hp10BII: P/YR=12, X P/YR=30, NOM%=7.35, PV=160,000, Press PMT (Answer: $1,102.36)

C. Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$160,000	
Payment	PMT		$1,102.36
Future Value	FV	$0	
Interest Rate	I	7.35%	

Click here to return to Problem 1 (a)

Problem 1 (b) Sensitivity Analysis

Sensitivity Analysis. Recall that you are seeking to buy an "entry-level" house that is available from the builder for $200,000. Currently, you can get an 80% loan, at 7.35% with a 30 year amortization. Now let's run through some calculations and solve for the correct answers. (Note: for each question, go back to initial assumptions for the other variables).

(1) Interest rates fall to 6%, what are mortgage payments?

Using the HP 10BII

hp10BII: P/YR=12, xP/YR:N=30, NOM%=6, PV=160,000, Press PMT (Answer: $959.28)

Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$160,000	
Payment	PMT		$959.28
Future Value	FV	$0	
Interest Rate	I	6.00%	

Return to Question 1-b (1)

(2) You add $20,000 for landscaping and other amenities and build that into the price of the house. What is your mortgage payment? Note: new initial loan is:

($200,000 + $20,000) x 80% = $176,000

Using the HP 10BII

P/YR=12, xP/YR:N=30, NOM%=7.35, PV=176,000, Press PMT (Answer: $1,212.59)

Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$176,000	
Payment	PMT		$1,212.59
Future Value	FV	$0	
Interest Rate	I	7.35%	

Return to Question 1-b (1)

(3) The term of the loan is reduced to 25 years. What is your mortgage payment?

Using the HP 10BII

P/YR=12, xP/YR:N=25, NOM%=7.35, PV=160,000, Press PMT (Answer: $1,166.82)

Excel Approach

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	25	
Present Value	PV	$160,000	
Payment	PMT		$1,166.82
Future Value	FV	$0	
Interest Rate	I	7.35%	

Return to Question 1-b (3)

(4) Assume you sell the house after 5 years. How much will you still owe on the initial mortgage?

Using the HP 10BII

Step 1 P/YR=12, xP/YR:N=30, NOM%=7.35, PV=160,000, Press PMT
(Answer: $1,102.36)

Step 2 P/YR=12, xP/YR:N=25, NOM%=7.35, PMT=1,102.36, Press PV
(Answer: $151,161)

Excel Approach

Step 1

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$160,000	
Payment	PMT		$1,102.36
Future Value	FV	$0	
Interest Rate	I	7.35%	

Step 2

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	25	
Present Value	PV		$151,161.04
Payment	PMT	$1,102.36	
Future Value	FV	$0	
Interest Rate	I	7.35%	

Return to Question 1-b (4)

(5) Assume your house appreciates 6% per year, compounded monthly. How much equity will you walk away with if you sell after 10 years? (Note: ignore selling expenses)

This is a two step problem. First, calculate the gross future selling price of your house 10 years out. Second, calculate the outstanding principle remaining on the 30 year loan and net that against the future value of the house.

Using the HP 10BII

Step 1 P/YR=12, X P/YR=10, NOM%=6, PV=200,000, Press FV
(Answer: $363,879.35)

Step 2 P/YR=12, X P/YR=20, NOM%=7.35, PMT=1,102.36, Press PV
(Answer: $138,409.86)

So, what you walk away with is the difference: $363,879.35 - $138,409.86 = **$225,469.49**

Excel Approach

Step 1

FV1

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	10	
Present Value	PV	$200,000	
Payment	PMT	$0.00	
Future Value	FV		$363,879.35
Interest Rate	I	6.00%	

Step 2

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	20	
Present Value	PV		$138,409.86
Payment	PMT	$1,102.36	
Future Value	FV	$0	
Interest Rate	I	7.35%	

$363,879.35
($138,409.86)
$225,469.49

Return to Question 1-b (5)

(6) If your cost of capital is 8%, what did the deal in 1-b (5) actually cost you in PV?

This is another two-step problem. First, calculate the PV of the monthly payments you will actually have to make over the 10 years. Second, calculate PV of the net residual (i.e., sales price – principal balance).

Using the HP 10BII

Step 1 PV of Payment: P/YR=12, xP/YR:N=10, NOM%=8, PMT=1,102.36, Press PV
(Answer: $90,858.14)

Step 2 PV Net Reversion: P/YR=12, xP/YR:N=10, NOM%=8, FV=225,469.49, Press PV
(Answer: $101,579.29)

Step 3 Net off Initial Downpayment to get residual PV

$101,579.29 - $40,000 = $61,579.29

PV Cost = -$90,858.14 + $61,579.29 **= -$29,278.85**

Excel Approach

PVA Step 1

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	10	
Present Value	PV		$90,858.14
Payment	PMT	$1,102.36	
Future Value	FV	$0	
Interest Rate	I	8.00%	

Step 2

PV1

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	10	
Present Value	PV		$101,579.29
Payment	PMT	$0.00	

Future Value	FV	$225,469	
Interest Rate	I	8.00%	

Step 3 Net off Initial Downpayment to get residual PV

$101,579.29 - $40,000 = $61,579.29

PV Cost = -$90,858.14 + $61,579.29 **= -$29,278.85**

Note in this case, your cost of ownership is significantly reduced by the appreciation in your house. For example, in terms of capital returns, in PV terms, your $40,000 equity investment grows to $101,579.29 in PV terms for a net PV gain of $61,579.29. This "gain" offsets much of your mortgage costs (PV $90,858.14, and actual outlay of $132,283 ($1,102.36 x 120 mo.)). In this example, you live in the house for an "average" monthly cost of $244 ($29,278.85/120).

Return to Question 1-b (6)

Problem 2

a. Assume that in this case, you have decided to move out to a northern county in which the housing prices are more "affordable," and have found your dream house for a mere $180,000. Assuming you can get a loan for 80% of the value for 30 years, and interest rates are still 7.35%. What would your mortgage payments be per month?

Using the HP 10BII

Loan Amount = $180,000 x 80% (Answer: $144,000)

P/YR=12, xP/YR:N=30, NOM%=7.35, PV=144,000, Press PMT
(Answer: $992.12)

Excel Approach

PR

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$144,000	
Payment	PMT		$992.12
Future Value	FV	$0	
Interest Rate	I	7.35%	

Return to Question 2-a

b. As of January 24, 2001, the following mortgage terms were available:

Today's National Average Mortgage Rates from HSH

Program	Rate	Points
30 Year FRM	7.35%	0.85%
15 Year FRM	6.90%	0.82%
1 Year ARM	6.59%	0.88%

(1). What are the initial monthly payments under the 15 year and 1 year ARM options (ignore the points)? Note: the ARM is a 30 year amortization.

Using the HP 10BII

15 year P/YR=12, xP/YR:N=15, NOM%=6.9, PV=144,000, Press PMT
(Answer: $1,286.28)

1 year ARM P/YR=12, xP/YR:N=30, NOM%=6.59, PV=144,000, Press PMT
(Answer: $918.72)

Excel Approach

15 Year Fixed:

PR

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	15	
Present Value	PV	$144,000	

Payment	PMT		$1,286.28
Future Value	FV	$0	
Interest Rate	I	6.90%	

1 Year Arm

PR

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV	$144,000	
Payment	PMT		$918.72
Future Value	FV	$0	
Interest Rate	I	6.59%	

Click here for Question 2-b (1)

(2). Compare the "costs" of the 30 year and the15 year mortgages above. Which one is "more affordable?" Assume your cost of capital is 10% annual, compounded monthly.

In some cases, the answer to this question depends on your definition of "affordable;" either the lowest PV cost, or the lowest monthly payment. As noted below, in this case the 30 year loan has both a lower PV cost ($6,645.03), and a lower monthly payment.

Using the HP 10BII

30 year P/YR=12, xP/YR:N=30, NOM%=10, PMT=992.12, Press PV
(Answer: $113,052.89)

15 year P/YR=12, xP/YR:N=15, NOM%=10, PMT=1,286.28, Press PMT
(Answer: $119,697.92)

Excel Approach

30 year

PVA

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	30	
Present Value	PV		$113,052.89
Payment	PMT	$992.12	
Future Value	FV	$0	
Interest Rate	I	10.00%	

15 year

PVA

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	15	
Present Value	PV		$119,697.92
Payment	PMT	$1,286.28	
Future Value	FV	$0	
Interest Rate	I	10.00%	

Click here for Question 2-b (2)

(3) Let's build on the prior question. What rate would the 15 Year loan have to be at to make you indifferent in terms of the present value cost vs. the 30 year loan? Refer to the answer to 2-b (2) to make sure you have the right inputs.

As noted in the solution below, you have a two-step problem again; first solve for the rate, and then solve for the payment under that rate. To check your answer, calculate the PV of the new payment; what do you think it should be before you run the numbers?

Using the HP 10BII

Step 1. Calculate rate which balances PMT on 15 year with PV on 30 year:

15 year P/YR=12, xP/YR:N=15, PMT=992.12, PV=113,052.89, +/-, Press I/YR (Answer: 6.62%)

Step 2. Test PV on the 15 year which equals the 30 PV of $113,053

15 year P/YR=12, xP/YR:N=15, NOM%=6.62, PMT=992.12, Press PV (Answer: $113,053)

Excel Approach

Step 1. Calculate required rate for 15 year PMT to equal 30 PV:

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	15	
Present Value	PV	$113,053	
Payment	PMT	$992.12	
Future Value	FV	$0	
Interest Rate	I		6.62%

=RATE(12*15,-992.12,113053)*12

RATE(nper, pmt, pv, [fv], [type], [guess])

Step 2. Test Result: PV15 should = PV 30

PVA

Factor	Code	Initial	Answer
Compounding/Period	m	12	
Term	t	15	
Present Value	PV		$113,034.06

Payment	PMT	$992.12	
Future Value	FV	$0	
Interest Rate	I	6.62%	

Click here for Question 2-b (3)

Problem 3. Alternative Financing

a. To simplify the discussion, we will use the same basic problem presented in Problem 2. That is, your dream house costs $180,000, you can get a loan for 80% of the value for 30 years, and interest rates are still 7.35% annual, compounded monthly.

(1) Now, let's go back to your original assumptions. Assume that you can only afford to make $900 in monthly payments.The good news for you is that the housing market is softening and the builder is anxious to get you into the new house. How much would he have to "buy-down" your mortgage amount to allow you to afford the 30 year, fully amortizing loan at 7.35%?

Step 1 hp10BII: P/YR=12, X P/YR=30, NOM%=7.35, PMT=900, Press PV
(Answer: $130,629.38)

Step 2

Price	$180,000
x LV	80%
Eligible Loan	$144,000
Affordable Loan	$130,629
Buydown	$13,371

As noted, your $900 monthly will only support a $130,629 loan; the builder must make up the gap between what you can carry and what the lender requires to earn it's 7.35%. In effect, they will reduce your loan by $13,371 by giving you a rebate.

Click here for Question 3-a (1)

(2) Why would a builder use a rebate rather than just cutting the price of the house; wouldn't that be simpler?

The answer to this question could be based on a number of factors associated with why the builder would be reluctant to merely reduce the price of the house. First, an outright price reduction could taint the subdivision/development, calling attention to a basic disconnect between asking prices and the actual

market for the houses. Second, the building would be undermining the confidence of his previous buyers, causing some to question the true worth and/or profit margins on their house. Third, the builder would be creating an adverse appraisal bias that could undermine the value of subsequent or prior houses. That is, an appraiser would use comparable sales as evidence of market value. Unless the price cut is somehow disguised, the reduced price could be the ceiling for subsequent sales. Fifth, the builder might have some minimum pricing criteria for the development and/or construction lender that could cause a breach of a loan covenant. Finally, the builder might want to close out the development and move into market segments that are not as interest-rate dependent for a next venture.

Click here for Question 3-a (2)

(3) Assume that the builder had a construction loan that carried a 200 basis point spread over the rate on a fully amortizing, 30 year permanent loan. If he had to carry the house for 2 years until the market would absorb the house, how much more would he have to charge then to break even? (Note: the builder has financed 90% of the actual costs and has a 15% margin built into the price).). He also considers his cost of capital 15%.

Answer: As in most TVM problems, this question involves several steps. First, you must calculate the amount of the construction loan ($137,700). Second, calculate the interest-only payments to carry that loan ($1,072.91). Third, find the future value of those payments using the builder's 15% cost of capital. Fourth, gross up the cost of the loan carry, and add that to the initial price.

Step 1: Amount of Construction loan

Sales Price		$180,000		
x 1 - Margin		85%		
Actual Cost		$153,000		
Construction LV		90%	Step 2: Interest-only payments	
Construction Loan		$137,700	x 9.35%/12 =	$1,072.91

Step 2 hp10BII: P/YR=12, X P/YR=2, NOM%=15, PMT=1,072.91, Press FV
(Answer: $29,814.11)

Step 3

FV cost	$29,814
/ 1- Margin	85%
Gross FV	$35,075
Current Price	$180,000
Future Price	$215,075

Click here for Question 3-a (3)

(4) Let's say the builder didn't want to carry the property to wait out the market, what discount would that translate to if the builder decided to cut the cost today to move the property?

hp10BII: P/YR=12, X P/YR=2, NOM%=15, FV=215,075, Press PV
(Answer: $159,628.03)

Click here for Question 3-a (4)

Click here to go to beginning.